No. 2. Example: Given $x^2-19y^2=1$, to find x and y.

Value	N	m^2	n^2+Diff.	n^2-Diff.
19	$=19\times$	1^2	$=4^2+3$	$=5^2-6$
76	$=19\times$	2^2	$=8^2+12$	$=9^2-5$
171	$=19\times$	3^2	$=13^2+2$	

This gives at once $\left.\begin{array}{r} m=3 \\ n=13 \\ \text{Diff.}\quad +2 \end{array}\right\}$ and in(A) we have $y=39$, $x=180$.

Proof: $28900-19\times1521=1$.

No. 3. Example: $x^2-31y^2=1$, to find x and y.

Value N m^2 n^2+ Diff. n^2-Diff. Terms No. 1 2 3

$$\left.\begin{array}{l} 31=31\times1=5^2+\ 6^2=6-5 \\ 124=31\times2^2=11^2+\ 3 \\ 279=31\times3^2=\dots\dots\dots\dots 17^2-10 \\ 496=31\times4^2=22^2+12 \\ 775=31\times5^2=\dots\dots\dots\dots 28^2-9 \end{array}\right\} \begin{array}{lrrr} & & & +5 \\ \text{Take } m= & 3 & +\ 2 & 7 \\ & & & +28 \\ n= & 17 & +\ 11 & 39 \\ \text{Diff.} & -10 & +3 & -2 \\ D_1 & & +13 & -5 \\ D_2 & & -18\dots & -18 \end{array} \left.\right\} \text{give } \begin{array}{l} y=273 \\ x=1520 \end{array}$$

(TO BE CONTINUED.)

1. **Proposed by EARL D. WEST, West Middleburg, Logan County, Ohio.**

It is required to divide a given square number into two such parts that each part will be a square number.

I. **Solution by Hon. JOSIAH H. DRUMMOND, LL. D., Portland, Maine.**

No general solution can be given because there are *comparatively* few square numbers which are severally equal to the sum of two other squares. The well known equation $(2pq)^2+(p^2-q^2)^2=(p^2+q^2)^2$, in which $2pq$, p^2-q^2, and p^2+q^2 represent the sides of right triangles, is believed to include the cases in which the sum of two squares equals a third square. If this is true, it follows that the square root of the given square $(p^2+q^2)^2$ must also be the sum of two squares, p^2+q^2; and we have $p^2+q^2=$the square root of the given number to find $(2pq)^2$ and p^2-q^2, p being $>q$. If this can be done only by trial let us determine the limits within which the trials may be confined. Let m be the square root of the given number, and p and q may be any part positive numbers the sum of whose squares$=m$. Also, if any factor of m is equal to the sum of two squares, say s^2 and t^2, p and q may be s and t multiplied, respectively, by the corresponding factor, but, if any value of p is not prime to the corresponding value of q, the value obtained by some of the processes may be the same as the value obtained by another process. If $p^2+q^2=m$, remembering that $p>q$, we have $p<\sqrt{m}$ and $p<\sqrt{\frac{m}{2}}$. Take $m=82$, then $p<10$ and $p>6$; that is, p may be 7, 8, or 9, but $=7$ or 8 gives impossible values for q; while $p=9$, gives $q=1$. Then $2pq=18$ and $(p^2-q^2)=80$. Hence 18^2 and 80^2 are the numbers, and we have $18^2+80^2=82^2$. But $82=2\times41$; then $s<7$ and $s>4$, and so may be 5

or 6; but $s=6$ makes t impossible, while $s=5$ makes $t=4$, and we have $2st=40$ and $(s^2-t^2)=18$. So $2pq=80$ and $(p^2-q^2)=18$, as before. Take $m=50$, p may be 6 or 7, but 6 makes q impossible, while $p=7$ gives $q=1$, and $2pq=14$ and $p-q^2=48$. $\therefore$ $14^2+48^2=50^2$. But $50=5\times10$, and $5=4+1$; then $s=2$ and $t=1$, $2st=4$, and $s^2-t^2=3$, $2pq=40$, and $p^2-q^2=30$, and $30^2+40^2=50^2$. Solutions may be obtained from other factors, but they give one of these two results. If $m=65$, it may be shown in the same manner that there are 4 pairs of numbers answering the conditions.

II. Solution by R. J. ADCOCK, Larchland, Illinois.

Since $(x^2+y^2+z^2+u^2+v^2)^2=(x^2+y^2+z^2+u^2+v^2)^2+(2xv)^2+(2yv)^2+(2zv)^2+(2uv)^2$, is true for two or any greater number of letters. Then

$$\left[(2x+1)^2+(2y)^2\right]^2=\left[(2x+1)^2-(2y)^2\right]^2+\left[2(2x+1)2y\right]^2.$$

$$\text{And, } 1=\left[\frac{(2x+1)^2-(2y)^2}{(2x+1)^2+(2y)^2}\right]^2+\left[\frac{2(2x+1)2y}{(2x+1)^2+(2y)^2}\right]^2.$$

If n^2 be the given square number,

$$n^2=\left[\frac{(2x+1)^2-(2y)^2}{(2x+1)^2+(2y)^2}\times n\right]^2+\left[\frac{2(2x+1)2yn}{(2x+1)^2+(2y)^2}\right]^2$$ are the two square parts into which n^2 is divided. Observing that x may have any value including 0, y any value not $=0$, and in order to avoid repetion of parts into which the square number is divided, the numbers for x and y must not make $2x+1$ and $2y$ have a common factor or be equal. If $x=0$, and $y=1$, the two square parts of n^2 are $\left(\frac{3n}{5}\right)^2$ and $\left(\frac{4n}{5}\right)^2$.

Also solved by *H. W. Draughon*, *G. B. M. Zerr*, *P. S. Berg*, *A. L. Foote*, *P. H Philbrick*, and *H. C. Whitaker*.

2. Proposed by J. M. COLAW, Principal of High School, Monterey, Virginia.

Find two numbers, such that the difference of their squares may be a cube, and the difference of their cubes a square.

I. Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let $6x^3$, $10x^3$ be the numbers.

Then $100x^6-36x^6=64x^6=$ a cube number, and $1000x^9-216x^9=784x^9=(28)^2x^9=$ a square number, when x is a perfect square. Let $x=1$, then 6 and 10 are the numbers. Other values can be obtained by substituting 4, 9, 16, etc. for x.

II Solution by C. W. M. BLACK, Department of Mathematics in Wilmington Conference Academy, Dover, Delaware.

(A). Represent the numbers by x, and $x+a$. The difference of the cubes will be (1), $a(3x^2+3ax+a^2)$, and the difference of the squares (2), $a(2x+a)$. For one solution the first will be a square if $a=3x^2+3ax+a^2$, and the second a cube if $a=(a+2x)^2$. Solving these equations, $a=1$ and $x=-1$ or 0, $a=0$ and $x=0$, making the required numbers -1 and 0, 0 and 1, or 0 and 0. Also, $a(2x+a)$ will be a cube if (3), $a^2=a+2x$. Combining (1) and (3), $(a-1)(3a^2+3a+4)=0$. The first factor gives $a=1$, whence $x=-1$ or 0, as before. The second factor gives an imaginary result. As these results are not satisfactory, we seek another method for finding other values, if there are any.

(B). It may easily be shown that if x and $x+a$ are positive intergers